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Non-factorizable effects in $B - \overline{B}$ mixing

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Abstract. We study the B-parameter ("bag factor") for $B - \overline{B}$ mixing within a recently developed heavy-light chiral quark model. Non-factorizable contributions in terms of gluon condensates and chiral corrections are calculated. In addition, we also consider $1/m_Q$ corrections within heavy quark effective field theory. Perturbative QCD effects below $\mu = m_b$ known from other work are also included. Considering two sets of input parameters, we find that the renormalization invariant B-parameter is $\hat{B} = 1.51 \pm 0.09$ for B_d and $\hat{B} = 1.40 \pm 0.16$ for B_s.

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1 Introduction

Studies of the neutral K-meson system have played a major role in modern particle physics [1]. Because of weak interactions, a neutral K meson may be converted to a neutral $\overline{K}$ meson. This process, known as $K - \overline{K}$ mixing, determines both the mass-difference between the physical neutral states K_L and K_S and the dominating CP-violating effect in neutral K-meson decays to pions (the ε-effect). The neutral B-meson system has rather similar properties as the neutral K-system. The difference when going to $B - \overline{B}$ mixing is the importance of other KM quark mixing factors and other mass scales, in particular the B-mesons are about ten times heavier than the K-mesons.

In general, non-leptonic processes may be described by an effective Lagrangian which is a linear combination of quark operators. The (Wilson) coefficients of the operators can be calculated in perturbation theory combined with the renormalization group equations [2]. At quark level, the leading order diagrams for $B - \overline{B}$ mixing are given by the so called box diagram. This diagram has double W-exchange between two quark lines, and generates an effective Lagrangian(Hamiltonian) for the quark transition $\bar{b}d \rightarrow \bar{d}b$. This Lagrangian has (for all practical purposes) only *one* operator times a Wilson coefficient containing the effects of the virtual (u, c, t) quarks running in the loop. This Wilson coefficient has also been corrected for perturbative QCD effects within the renormalization group equations. Such calculations has been performed to next to leading order. For $B_s - \overline{B_s}$ mixing one considers the corresponding $\bar{b}s \rightarrow \bar{s}b$ transition.

The difficult part is to calculate the matrix elements of the quark operators between the mesonic states, which is a non-perturbative issue. This has been done by lattice simulations [3, 4] or by quark models [5]. The hadronic matrix element is, as for $K-\overline{K}$ mixing, parameterized through the so called B- ("bag"-) parameter which is by construction equal to one in the naive limit when vacuum states are inserted between the quark currents in the $B-\overline{B}$ mixing operator.

In a previous paper [6], $K-\overline{K}$ mixing was calculated within a chiral quark model (χQM) combined with chiral perturbation theory. Within the χQM, non-factorizable contributions can also be calculated in terms of gluon condensates. The purpose of this paper is to perform a similar analysis for $B-\overline{B}$ mixing. We are using a recently developed heavy-light chiral quark model (HLχQM) [7], where non-factorizable effects can be incorporated by means of gluon condensates and chiral loops.

2 $B-\overline{B}$ mixing and heavy quark effective theory

At quark level, the standard effective Lagrangian describing $B-\overline{B}$ mixing is [2]:

$$\mathcal{L}_{eff}^{\Delta B=2} \; = \; -\frac{G_F^2}{4\pi^2} M_W^2 \left(V_{tb}^* V_{tq}\right)^2 S_0\left(x_t\right) \eta_B \, b(\mu) \, Q(\Delta B=2) \; , \tag{1}$$

where G_F is Fermi's coupling constant, the V's are KM factors [8] (for which $q=d$ or s for B_d and B_s respectively) and S_0 is the Inami-Lim function [9] due to short distance electroweak loop effects for the box diagram:

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \operatorname{Log} x}{2(1-x)^3} \, . \tag{2}$$

In our case, $x = x_t \equiv m_t^2/M_W^2$, where m_t is the top quark mass. Because of its large mass, the top quark gives the dominant contribution. Also the u and c quarks are running in the loop, but these contributions are KM suppressed. The quantity $Q(\Delta B=2)$ is a four quark operator:

$$Q(\Delta B=2) = \overline{q_L} \, \gamma^\alpha \, b_L \, \overline{q_L} \, \gamma_\alpha \, b_L \; , \tag{3}$$

where q_L (b_L) is the left-handed projection of the q (b)-quark field. The quantities η_B and $b(\mu)$ are calculated in perturbative quantum chromodynamics (QCD). At the next to leading order (NLO) analysis it is found that $\eta_B = 0.55 \pm 0.01$ [2]. Furthermore, for a renormalization point μ in perturbative QCD equal to or below m_b,

$$b(\mu) \; = \; \left[\alpha_s(\mu)\right]^{-6/23} \left[1 + \frac{\alpha_s(\mu)}{4\pi} J_5\right] \; , \tag{4}$$

where $J_5 = 1.63$ in the naive dimension regularization scheme (NDR). At $\mu = m_b$ ($= 4.8$ GeV) one has $b(m_b) \simeq 1.56$.

The matrix element of the operator $Q(\Delta B = 2)$ between the meson states is parameterized by the bag parameter B_{B_q}:

$$\langle B|Q(\Delta B = 2)|\overline{B}\rangle \equiv \frac{2}{3} f_B^2 M_B^2 B_{B_q}(\mu) \ . \tag{5}$$

By definition, $B_{B_q} = 1$ within *naive factorization*, also named vacuum saturation approach (VSA). This means to insert a vacuum state between the two heavy-light currents in the operator $Q(\Delta B = 2)$, and use the matrix elements defining the decay constant f_B:

$$\langle 0|\overline{q_L}\,\gamma^\mu\, b|\overline{B}(p)\rangle = \frac{i}{2}\, f_B\, p^\mu \qquad \text{and} \qquad \langle B(p)|\overline{q_L}\,\gamma^\mu\, b|0\rangle = -\frac{i}{2}\, f_B\, p^\mu \ . \tag{6}$$

One may combine naive factorization with the large N_c expansion, where N_c is the number of colours. Then one finds $B_{B_q} = 3(1 + 1/N_c)/4$, giving $B_{B_q} = 3/4$ in the (naive) large N_c limit. We will see later that there are important non-factorizable contributions of order $1/N_c$. In general, the matrix elements of the operator $Q(\Delta B = 2)$ are dependent on the renormalization scale μ, and thereby B_{B_q} depends on μ. As for $K - \overline{K}$ mixing, one defines a renormalization scale independent quantity

$$\hat{B}_{B_q} \equiv b(\mu) B_{B_q}(\mu) \ . \tag{7}$$

Within lattice gauge theory, values for $\hat{B}_{B_q}$ between 1.3 and 1.5 are obtained [3, 4].

The mass difference between the weak eigenstates (B_H and B_L) are related to the bag parameter in the following way for $B_q = B_d, B_s$:

$$\Delta m_q = \frac{G_F^2}{6\pi^2} m_{B_q} f_{B_q}^2 \hat{B}_{B_q} \eta_B M_W^2\, S_0\left(m_t^2/M_W^2\right)\, |V_{tq}^* V_{tb}|^2 \ . \tag{8}$$

In order to extract the KM matrix elements it is crucial to have a precise knowledge of the bag parameter $\hat{B}_{B_q}$, and the weak decay constant f_{B_q}.

The b -quark is heavy compared to the typical hadronic scale of order 1 GeV, where confinement and chiral symmetry breaking effects are essential. Perturbative effects below the b-quark scale may then be calculated down to 1 GeV by means of heavy quark effective theory (HQEFT. See [10] for a review). Thus HQEFT also allows us to evolve the matrix element (3) from $\mu = m_b$ down to 1 GeV.

HQEFT is a systematic expansion in $1/m_b$. The heavy quark field $b(x)$ is replaced by a "reduced" field, $Q_v^{(+)}(x)$ or $Q_v^{(-)}(x)$, which is related to the full field the in following way:

$$Q_v^{(\pm)}(x) = P_\pm e^{\mp i m_b v\cdot x} b(x) \,, \tag{9}$$

where $P_\pm$ are projecting operators $P_\pm = (1 \pm \gamma \cdot v)/2$. The reduced field $Q_v^{(+)}$ can only annihilate heavy quarks. In order to describe heavy anti-quarks one has to use $Q_v^{(-)}$. In other words, $Q_v^{(+)}(Q_v^{(-)})$ annihilates (creates) a heavy quark (anti-quark) with velocity v. The Lagrangian for heavy quarks is ($Q_v = Q_v^{(\pm)}$):

$$\mathcal{L}_{\mathrm{HQEFT}} = \pm\overline{Q_v}\, iv\cdot D\, Q_v + \frac{1}{2m_Q}\overline{Q_v}\,\left(-C_M\frac{g_s}{2}\sigma\cdot G + (iD_\perp)^2_{\mathrm{eff}}\right) Q_v + \mathcal{O}(m_Q^{-2}) \ , \tag{10}$$

where D_μ is the covariant derivative containing the gluon field (eventually also the photon field), and $\sigma\cdot G = \sigma^{\mu\nu}G^a_{\mu\nu}t^a$, where $\sigma^{\mu\nu} = i[\gamma^\mu,\gamma^\nu]/2$, $G^a_{\mu\nu}$ is the gluonic field tensors, and t^a are the colour matrices. This chromo-magnetic term has a factor C_M which is one at tree level, but slightly modified by perturbative QCD effects below the scale m_b . It has been calculated to NLO [12, 13]. Furthermore, $(iD_\perp)^2_{\mathrm{eff}} = C_D(iD)^2 - C_K(iv\cdot D)^2$. At tree level, $C_D = C_K = 1$. Here, C_D is not modified by perturbative QCD, while C_K is different from one due to perturbative QCD corrections [11]. In our case, $m_Q = m_b$ is the heavy quark mass.

Running from $\mu = m_b$ down to $\mu = \Lambda_\chi = 1$ GeV, there will appear more operators. Some stem from the heavy quark expansion itself and some are generated by perturbative QCD effects. The $\Delta B = 2$ operator in Eq. (3) for $\Lambda_\chi < \mu < m_b$ can be written [14, 15, 16]:

$$\begin{aligned} Q(\Delta B = 2) = \quad & C_1(\mu)\, Q_1 + C_2(\mu)\, Q_2 + \frac{1}{m_b}\left(\sum_i^6 a_i(\mu)S_i(\mu)\right. \\ & \left. + \sum_i^3 h_i(\mu)X_i(\mu)\right) + \mathcal{O}(1/m_b^2) \ . \end{aligned} \tag{11}$$

The operator Q_1 is $Q(\Delta B = 2)$ for b replaced by $Q_v^{(\pm)}$, while Q_2 is generated within perturbative QCD for $\mu < m_b$. The operators S_i and X_i are taking care of $1/m_b$ corrections. The quantities C_1, C_2, a_i, h_i are Wilson coefficients. ($C_1 = 1+\mathcal{O}(\alpha_s)$ and $C_2 = 0+\mathcal{O}(\alpha_s)$). The explicit expressions for the operators are

$$Q_1 = 2\,\overline{q_L}\,\gamma^\mu\, Q_v^{(+)}\ \ \overline{q_L}\,\gamma_\mu\, Q_v^{(-)} \ , \tag{12}$$

$$Q_2 = 2\,\overline{q_L}\,v^\mu\, Q_v^{(+)}\ \ \overline{q_L}\,v_\mu\, Q_v^{(-)} \ , \tag{13}$$

$$\begin{aligned} X_1 = \; & 2\,\overline{q_L}\, i\not{D}\, Q_v^{(+)}\ \ \overline{q_L}\, Q_v^{(-)} + 2\,\overline{q_L}\, iD^\mu\, Q_v^{(+)}\ \ \overline{q_L}\,\gamma_\mu\, Q_v^{(-)} \\ & -2\, i\,\varepsilon_{\lambda\mu\nu\rho}\, v^\lambda\,\overline{q_L}\, iD^\mu\gamma^\nu\, Q_v^{(+)}\ \ \overline{q_L}\,\gamma^\rho\, Q_v^{(-)} \\ & +2\,\overline{q_L}\, Q_v^{(+)}\ \ \overline{q_L}\, i\not{D}\, Q_v^{(-)} + 2\,\overline{q_L}\,\gamma_\mu\, Q_v^{(+)}\ \ \overline{q_L}\, iD^\mu\, Q_v^{(-)} \\ & -i2\ \varepsilon_{\lambda\mu\nu\rho}\, v^\lambda\,\overline{q_L}\,\gamma^\nu\, Q_v^{(+)}\ \ \overline{q_L}\, iD^\mu\gamma^\rho\, Q_v^{(-)} \ , \end{aligned} \tag{14}$$

$$X_2 = 8\,\left[iv\cdot\partial(\overline{q_L}\, Q_v^{(+)})\right]\overline{q_L}\, Q_v^{(-)} + 2\,\left[iv\cdot\partial(\overline{q_L}\gamma_\mu\, Q_v^{(+)})\right]\overline{q_L}\,\gamma^\mu\, Q_v^{(-)} \ , \tag{15}$$

$$X_3 = 4\,\left[iv\cdot\partial(\overline{q_L}\gamma_\mu\, Q_v^{(+)})\right]\overline{q_L}\,\gamma^\mu\, Q_v^{(-)} \ . \tag{16}$$

The operators S_i are nonlocal and are combinations of the leading order operators $Q_{1,2}$ and a term of order $1/m_Q$ from the effective Lagrangian (10):

$$\begin{aligned}
\frac{S_1}{m_b} &= i\int dy^4 T\{Q_1(0), O_K(y)\}\ ,\\
\frac{S_2}{m_b} &= i\int d^4y T\{Q_2(0), O_K(y)\}\ ,\\
\frac{S_3}{m_b} &= i\int d^4y T\{Q_1(0), O_M(y)\}\ ,\\
\frac{S_4}{m_b} &= i\int d^4y T\{Q_2(0), O_M(y)\}\ ,
\end{aligned}\tag{17}$$

where

$$\begin{aligned}
O_K &\equiv \frac{1}{2m_b}\left(\overline{Q_v^{(+)}}\,(iD_\perp)^2_{\text{eff}}\,Q_v^{(+)} + \overline{Q_v^{(-)}}\,(iD_\perp)^2_{\text{eff}}\,Q_v^{(-)}\right)\ ,\\
O_M &\equiv -\frac{g_s}{4m_b}\left(\overline{Q_v^{(+)}}\,\sigma\cdot G\,Q_v^{(+)} + \overline{Q_v^{(-)}}\,\sigma\cdot G\,Q_v^{(-)}\right)\ ,
\end{aligned}\tag{18}$$

are the kinetic and magnetic operators of Eq. (10). There are no mixing between the local operators and the non-local operators, since the local operators do not need the non-local ones as counter-terms. The Wilson coefficients a_i will then be products of $C_{1,2}$ and $C_{M,K}$. The Wilson coefficients C_1 and C_2 have been calculated to NLO [14, 16] and for $\mu = \Lambda_\chi$, $C_1(\Lambda_\chi) = 1.22$ and $C_2(\Lambda_\chi) = -0.15$. The coefficients $h_{1,2,3}$ have been calculated to leading order (LO) in [15], and the result at $\mu = \Lambda_\chi$ is $h_1 = 0.52$, $h_2 = -0.16$ and $h_3 = -0.15$.

3 The heavy-light chiral quark model

In order to calculate the matrix elements we will use the heavy-light chiral quark model (HLχQM) recently developed in [7]. This is a type of quark loop model [17, 18, 19, 20] where the quarks couples directly to the mesons at the scale of chiral symmetry breaking Λ_χ, which we put equal to 1 GeV. What makes our model [7] distinct from other similar models is that it incorporates soft gluon effects in terms of the gluon condensate with lowest dimension [6, 21, 22, 23, 24]. The term in the Lagrangian describing this interaction can be obtained as a mean-field approximation of the (extended) Nambu-Jona-Lasinio model (NJL) [25, 20].

In this section we will give a short presentation of the HLχQM. In the next section we will use the model [7] to calculate non-factorizable soft gluon effects in $B - \overline{B}$ mixing.

The Lagrangian for the HLχQM is

$$\mathcal{L}_{\text{HL}\chi\text{QM}} = \mathcal{L}_{\text{HQEFT}} + \mathcal{L}_{\chi\text{QM}} + \mathcal{L}_{\text{Int}}\ .\tag{19}$$

The first term is given in Eq. (10). The light quark sector is described by the chiral quark model (χQM), having a standard QCD term and a term describing interactions between quarks and (Goldstone) mesons:

$$\mathcal{L}_{\chi\text{QM}} = \overline{\chi}\left[\gamma^\mu(iD_\mu + \mathcal{V}_\mu + \gamma_5\mathcal{A}_\mu) - m\right]\chi - \overline{\chi}\,\widetilde{M_q}\,\chi\ ,\tag{20}$$

where $\chi_{L,R}$ are the flavour rotated quark fields given by:

$$\chi_L = \xi^\dagger q_L \quad ; \qquad \chi_R = \xi q_R \quad ; \qquad \xi \cdot \xi = \Sigma \; . \tag{21}$$

where $q^T = (u, d, s)$ are the light quark fields. The left- and right-handed projections q_L and q_R are transforming after $SU(3)_L$ and $SU(3)_R$ respectively. The quantity ξ is a 3 by 3 matrix containing the (would be) Goldstone octet (π, K, η):

$$\xi = e^{i\Pi/f} \quad \text{where} \quad \Pi = \frac{\lambda^a}{2}\phi^a(x) = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8 \end{bmatrix} , \tag{22}$$

where f is the bare pion decay constant. In (20), m is the ($SU(3)$ - invariant) constituent quark mass for light quarks, and $\widetilde{M_q}$ contains the current quark mass matrix $\mathcal{M}_q$ and the field ξ:

$$\widetilde{M_q} \equiv \widetilde{M_q^V} + \widetilde{M_q^A}\gamma_5 \; , \quad \text{where} \tag{23}$$

$$\widetilde{M_q^V} \equiv \tfrac{1}{2}(\xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger + \xi \mathcal{M}_q \xi) \quad \text{and} \quad \widetilde{M_q^A} \equiv -\tfrac{1}{2}(\xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger - \xi \mathcal{M}_q \xi) \; . \tag{24}$$

The vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ in (20) are given by:

$$\mathcal{V}_\mu \equiv \frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \qquad ; \qquad \mathcal{A}_\mu \equiv -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \; . \tag{25}$$

Furthermore, the covariant derivative D_μ in (20) contains the soft gluon field forming the gluon condensates. The gluon condensate contributions are calculated by Feynman diagram techniques as in [6, 7, 22, 23]. They may also be calculated by means of heat kernel techniques as in [21, 25, 26].

The interaction between heavy meson fields and heavy quarks are described by the following Lagrangian:

$$\mathcal{L}_{Int} = -G_H \left[\overline{\chi}_a \, \overline{H_a^{(\pm)}} \, Q_v^{(\pm)} + \overline{Q_v^{(\pm)}} \, H_a^{(\pm)} \, \chi_a\right] + \frac{1}{2G_3} Tr\left[\overline{H_v^a} \, H_v^a\right] \; , \tag{26}$$

where G_H and G_3 are coupling constants and $H_a^{(\pm)}$ is the heavy meson field containing a spin zero and spin one boson:

$$H_a^{(\pm)} \equiv P_\pm (P_{a\mu}^{(\pm)} \gamma^\mu - i P_{5a}^{(\pm)} \gamma_5) \quad , \quad \overline{H_a^{(\pm)}} \equiv \gamma^0 (H_a^{(\pm)})^\dagger \gamma^0 \; . \tag{27}$$

The fields $P^{(+)}(P^{(-)})$ annihilates (creates) a heavy meson containing a heavy quark (anti quark) with velocity v.

Integrating out the quarks by using (10), (20) and (26), the effective Lagrangian up to $\mathcal{O}(m_Q^{-1})$ can be written as [27, 7]:

$$\mathcal{L} = \mp Tr\left[\overline{H_a^{(\pm)}} \, (iv \cdot \mathcal{D}_{ba} - \Delta_Q) \, H_b^{(\pm)}\right] - g_{\mathcal{A}} \, Tr\left[\overline{H_a^{(\pm)}} H_b^{(\pm)} \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu\right] , \tag{28}$$

where $i\mathcal{D}_{ba}^\mu = i\delta_{ba} D^\mu - \mathcal{V}_{ba}^\mu$. The term proportional to the quark-meson mass difference $\Delta_Q = M_H - m_Q$ in (28) is irrelevant for us due to the reparametrization invariance [10]. Also, it does not enter our loop integrals because our heavy meson fields are attached to our quark loops at zero external momentum. (The external momentum includes the piece $v^\mu \Delta_Q$). As shown in [7], the term $\sim 1/G_3$ in (26) is related to Δ_Q, and this term is also irrelevant within the present paper.

To obtain (28) from the HLχQM one encounters divergent loop integrals, which might be quadratic-, linear- and logarithmic divergent. For the kinetic term in (28) we obtain the identification:

$$-iG_H^2 N_c \left(I_{3/2} + 2mI_2 - i\frac{(3\pi-8)}{384m^3 N_c}\langle\frac{\alpha_s}{\pi}G^2\rangle \right) = 1 \ , \tag{29}$$

where $I_{3/2}$ and I_2 are the linear and logarithmic divergent integrals respectively, and $\langle\frac{\alpha_s}{\pi}G^2\rangle$ is the gluon condensate. To obtain the axial vector term proportional to $g_{\mathcal{A}}$, we obtain a similar condition, and combining it with (29), we obtain for the axial vector term

$$g_{\mathcal{A}} = 1 + \frac{4}{3} i G_H^2 N_c \left(I_{3/2} - \frac{im}{16\pi} \right) \ , \tag{30}$$

such that the (formally) linear divergent integral $I_{3/2}$ is related to the strong axial coupling $g_{\mathcal{A}}$ (or strictly speaking, its deviation from one). Analogously, within the pure light quark sector (the χQM), it is well known that the quadratic and logarithmic divergent integrals are related to the quark condensate and the bare decay constant f, respectively [17, 21, 22, 23, 26]:

$$\langle\, \overline{q}q \,\rangle \ = -4imN_c I_1 - \frac{1}{12m}\langle\frac{\alpha_s}{\pi}G^2\rangle \ , \tag{31}$$

$$f^2 \ = -i4m^2 N_c I_2 + \frac{1}{24m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle \ . \tag{32}$$

The divergent integrals I_1, I_2 and $I_{3/2}$ are listed in appendix A. The effective coupling G_H describing the interaction between the quarks and heavy mesons can be expressed in terms of m, f, $g_{\mathcal{A}}$, and the mass splitting between the 1^- state and 0^- state. Using (29), (30), (32) one finds a relation between this mass-splitting and the gluon condensate via the chromomagnetic interaction in (10) [7]:

$$\langle\frac{\alpha_s}{\pi}G^2\rangle = \frac{16f^2}{\pi\eta}\frac{\mu_G^2}{\rho} \ , \qquad G_H^2 = \frac{2m}{f^2}\rho \ , \qquad \eta \equiv \frac{(\pi+2)}{\pi} C_M(\Lambda_\chi) \ , \tag{33}$$

where

$$\rho \equiv \frac{(1+3g_{\mathcal{A}}) + \frac{\mu_G^2}{\eta\, m^2}}{4(1+\frac{N_c m^2}{8\pi f^2})} \quad , \quad \mu_G^2(H) \ = \ \frac{3}{2} m_Q (M_{H^*} - M_H). \tag{34}$$

In the limit where only the leading logarithmic integral I_2 is kept we obtain:

$$g_{\mathcal{A}} \to 1 \ , \qquad \rho \to 1 \ , \qquad G_H \ \to \ G_H^{(0)} \equiv \frac{\sqrt{2m}}{f} \ . \tag{35}$$

Note that $g_{\mathcal{A}} = 1$ is the non-relativistic value [27]. We observe that the mass-splitting between H and H^* sets the scale of the gluon condensate. This means that, while in [23] the gluon condensate was fitted to the $K \to (2\pi)_{I=2}$ amplitude, it is here determined in the strong sector alone (with a slightly lower value than in [23]).

The $1/m_Q$ corrections to the strong Lagrangian have been calculated in [7]. They may formally be put into spin dependent renormalization factors. This means that (28) is still valid with the replacement $H^{\rm r} = H\,(Z_H)^{-\frac{1}{2}}$, where Z_H and the renormalized (effective) coupling $\tilde{g}_{\mathcal{A}}$ are defined as:

$$Z_H^{-1} = 1 + \frac{\varepsilon_1 - 2d_M\varepsilon_2}{m_Q} \;, \tag{36}$$

$$\tilde{g}_{\mathcal{A}} = g_{\mathcal{A}}\left(1 - \frac{1}{m_Q}(\varepsilon_1 - 2d_{\mathcal{A}}\varepsilon_2)\right) - \frac{1}{m_Q}(g_1 - d_{\mathcal{A}}g_2) \;, \tag{37}$$

where

$$d_M = \begin{cases} 3 & \text{for} \quad 0^- \\ -1 & \text{for} \quad 1^- \end{cases} \qquad d_{\mathcal{A}} = \begin{cases} 1 & \text{for} \quad H^*H \quad \text{coupling} \\ -1 & \text{for} \quad H^*H^* \quad \text{coupling} \end{cases} \tag{38}$$

and:

$$\varepsilon_1 = -m + G_H^2\left(\frac{\langle\,\overline{q}q\,\rangle}{4m} + f^2 + \frac{N_c m^2}{16\pi} + \frac{C_K}{16}(\frac{\langle\,\overline{q}q\,\rangle}{m} - f^2) + \frac{1}{128m^2}(C_K + 8 - 3\pi)\langle\frac{\alpha_s}{\pi}G^2\rangle\right) \;, \tag{39}$$

$$g_1 = m - G_H^2\left(\frac{\langle\,\overline{q}q\,\rangle}{12m} + \frac{f^2}{6} + \frac{N_c m^2(3\pi + 4)}{48\pi} - \frac{C_K}{16}(\frac{\langle\,\overline{q}q\,\rangle}{m} + 3f^2) + \frac{1}{64m^2}(C_K - 2\pi)\langle\frac{\alpha_s}{\pi}G^2\rangle\right) \;, \tag{40}$$

$$g_2 = \frac{(\pi+4)}{(\pi+2)}\frac{\mu_G^2}{6m} \;. \tag{41}$$

4 Bosonizing $Q(\Delta B = 2)$

In this section we will discard $1/m_Q$ terms. We are then left with the operators $Q_{1,2}$ defined in Eq. (12) and (13). In order to find the matrix element of $Q_{1,2}$, one uses the following relation between the generators of $SU(3)_c$ (i, j, l, n are colour indices running from 1 to 3):

$$\delta_{ij}\delta_{ln} = \frac{1}{N_c}\delta_{in}\delta_{lj} \;+\; 2\,t^a_{in}\,t^a_{lj} \;, \tag{42}$$

where a is an index running over the eight gluon charges. This means that by means of a Fierz transformation, the operator Q_1 in (12) may also be written in the following way:

$$Q_1 = \frac{2}{N_c}\,\overline{q_L}\,\gamma^\mu\,Q_v^{(+)}\,\overline{q_L}\,\gamma_\mu\,Q_v^{(-)} \;+\; 4\,\overline{q_L}\,t^a\,\gamma^\mu\,Q_v^{(+)}\,\overline{q_L}\,t^a\,\gamma_\mu\,Q_v^{(-)} \;, \tag{43}$$

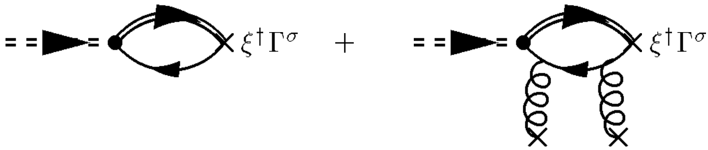


Fig. 1. Diagrams for bosonization of the left handed quark current

and similarly for Q_2.

The first (naive) step to calculate the matrix element of a four quark operator like Q_1 is by inserting vacuum states between the two currents. This vacuum insertion approach (VSA) corresponds to bosonizing the two currents in Q_1 and multiply them, as mentioned below Eq. (5). For one current, visualized in Fig. 1, one obtains [27, 7]:

$$\overline{q_L}\,\gamma^\mu\, Q_v^{(\pm)} \;\longrightarrow\; \frac{\alpha_H}{2} Tr\left[\xi_{hf}^\dagger \gamma^\alpha L\, H_h^{(\pm)}\right] \;, \tag{44}$$

Using the relations (29) - (32) for the divergent integrals, and also Eq. (33), we obtain [7]:

$$\alpha_H = \frac{G_H}{2}\left(-\frac{\langle\,\overline{q}q\,\rangle}{m} - 2f^2(1-\frac{1}{\rho}) + \frac{(\pi-2)}{16m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right) \;. \tag{45}$$

This bosonization has to be compared with the matrix elements defining the meson decay constant f_B given in Eq. (6). In those relations, b is the full quark field. Within HQEFT this matrix element will, below the renormalization scale $\mu = m_Q\,(= m_b)$, be modified in the following way:

$$\langle 0|\overline{q_L}\,\Gamma^\mu\, Q_v^{(+)}|\overline{B}(p)\rangle = \frac{i}{2}\, f_B\, M_B\, v^\mu$$

and

$$\langle B(p)|\overline{q_L}\,\Gamma^\mu\, Q_v^{(-)}|0\rangle = -\frac{i}{2}\, f_B\, M_B\, v^\mu \;, \tag{46}$$

where [10]

$$\Gamma^\mu \equiv C_\gamma(\mu)\,\gamma^\mu \,+\, C_v(\mu)\, v^\mu \;. \tag{47}$$

The coefficients $C_{\gamma,v}(\mu)$ are determined by QCD renormalization for $\mu < m_Q$. They have been calculated to NLO and the result is the same in MS and $\overline{MS}$ scheme [28]. In HLχQM the decay constant f_B can be calculated and the result is [7]:

$$\alpha_H = \frac{f_B\sqrt{M_B}}{C_\gamma(\mu)+C_v(\mu)} = \frac{f_{B^*}\sqrt{M_{B^*}}}{C_\gamma(\mu)} \;. \tag{48}$$

The second matrix element in (43) is genuinely non-factorizable, and we have to go beyond the VSA. However, in the approximation where only the lowest

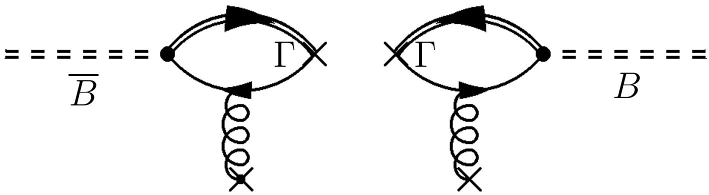


Fig. 2. Nonfactorizable contribution, $\Gamma \equiv t^a\,\gamma^\mu\,L$

gluon condensate is taken into account, the last term in (43) can be written in a *quasi-factorizable* way by bosonizating the heavy-light coloured current with an extra colour matrix t^a inserted and with an extra gluon emitted as shown in Fig. 2. Calculation of this diagram is straightforward when using the light quark propagator with just one soft gluon emitted:

$$S_G(k) \equiv \frac{g_s}{4} G^b_{\alpha\beta} t^b \left[\sigma^{\alpha\beta}(\gamma\cdot k + m) + (\gamma\cdot k + m)\sigma^{\alpha\beta}\right](k^2 - m^2)^{-2} \quad . \tag{49}$$

The part of the diagram to the left in Fig. 2 then gives the bosonized coloured current:

$$\left(\overline{q_L} t^a\,\gamma^\alpha Q_v^{(\pm)}\right)_{1G} \quad \longrightarrow \quad -\frac{G_H g_s}{8}\, G^a_{\mu\nu} \\ Tr\left[\xi^\dagger \gamma^\alpha L\, H^{(\pm)}\left(\pm i\, I_2\left\{\sigma^{\mu\nu}, \gamma\cdot v\right\} + \frac{1}{8\pi}\sigma^{\mu\nu}\right)\right] \tag{50}$$

where I_2 is to be identified with f^2 by the use of Eq. (32). The result for the right part of the diagram with $\bar{B}$ replaced by B is obtained by just changing the sign of v and letting $P_5^{(+)} \to P_5^{(-)}$ (remembering that $P_5^{(-)}$ *creates* a meson with a heavy anti quark). Multiplying the coloured currents, we obtain for the non-factorizable part of Q_1 and Q_2 to first order in the gluon condensate:

$$C_1\,\overline{q_L} t^a\,\gamma^\mu Q_v^{(+)}\;\overline{q_L} t^a\,\gamma_\mu\,Q_v^{(-)} + C_2\,\overline{q_L} t^a\,v^\mu Q_v^{(+)}\;\overline{q_L} t^a\,v_\mu\,Q_v^{(-)} \\ \to\; -\frac{\beta_B}{4}\langle\frac{\alpha_s}{\pi}G^2\rangle\left(C_1 P_{5i}^{(-)}\Sigma^\dagger_{ii} P_{5i}^{(+)} + (C_1 - \frac{1}{3}C_2)\,P_i^{(-)\mu}\Sigma^\dagger_{ii} P_{i\mu}^{(+)}\right) \quad , \tag{51}$$

where

$$\beta_B \equiv \frac{G_H^2}{128}\left\{1 + \frac{4\pi}{N_c}\left(\frac{f}{m}\right)^2 + \frac{8\pi^2}{N_c^2}\left(\frac{f}{m}\right)^4\right\} \quad , \tag{52}$$

and $\Sigma = \xi^2$, where ξ is given in Eq. (22). Note there is no sum over i, $i = 2, 3$ for $q = d, s$ respectively.

The Lagrangian in Eq. (20) contains couplings involving the the current mass term and the chiral quark fields. This makes it possible to calculate the counter-terms needed in order to keep the chiral Lagrangian finite after the inclusion of chiral loops. The counter-term for the factorizable part of the amplitude has

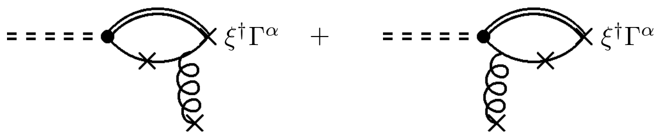


Fig. 3. Mass insertion in the nonfactorizable part of the current

been considered in [7] when calculating f_B. In the case of the non-factorizable part of the amplitude, we need to consider similar diagrams as those shown in Fig. 2, with mass insertion like in Fig. 3, where mass insertion is indicated by a cross on the light quark line. The bosonized current with mass insertion is

$$\left(\overline{q_L} t^a \gamma^\alpha Q_v^{(\pm)}\right)_{1G,m_q} \longrightarrow$$
$$\frac{G_H g_s}{32m\pi^2}\, \varepsilon^{\alpha\beta\mu\rho}(\pm v_\alpha) G^a_{\mu\rho}\, Tr\left[\xi^\dagger \gamma^\alpha L\, H_a^{(\pm)} \left(\widetilde{M}_q^V\right)_{aq} \gamma_\beta\gamma_5\right] . \quad (53)$$

This result can also be obtained by simply differentiating the right hand side of Eq. (50) with respect to m.

The bosonized version of the $Q(\Delta B = 2)$ operator can then be split in a pseudo scalar and a vector part:

$$Q(\Delta B = 2)_{\mathrm{Bos.}} = A_P\, P_{5i}^{(-)} \Sigma^\dagger_{ii} P_{5i}^{(+)} \;+\; A_V\, P_i^{(-)\mu} \Sigma^\dagger_{ii} P_{i\mu}^{(+)} \text{ , where:}$$
$$A_P = \frac{1}{2}(1+\frac{1}{N_c})(C_1 - C_2)\alpha_H^2 \left(1 + 2\frac{\omega_1}{\alpha_H} m_q\right) - C_1 \langle \frac{\alpha_s}{\pi} G^2\rangle \left(\beta_B + \omega_\beta m_q\right) \; , \quad (54)$$
$$A_V = \frac{1}{2}(1+\frac{1}{N_c}) C_1 \alpha_H^2 \left(1 + 2\frac{\omega_1}{\alpha_H} m_q\right) - \langle \frac{\alpha_s}{\pi} G^2\rangle \left((C_1 - \frac{C_2}{3})\beta_B + C_1 \omega_\beta m_q\right) .$$

The quantity ω_β is the counter-term obtained from (53), and ω_1 is a counter-term for f_{B_s} found in [7]:

$$\omega_\beta = \frac{G_H^2}{64\pi m}\left\{1 + \frac{4\pi f^2}{N_c m^2}\right\} , \quad (55)$$
$$\omega_1 = \frac{(1-3g_{\mathcal{A}})}{G_H} - \frac{(9\pi - 16)G_H}{192 m^3}\langle \frac{\alpha_s}{\pi} G^2\rangle . \quad (56)$$

For the current quark mass entering (54) we will use

$$m_d = -m_\pi^2 f^2 / \langle\, \overline{q} q\,\rangle \text{ , and } m_s = -m_K^2 f^2 / \langle\, \overline{q} q\,\rangle \; . \quad (57)$$

The term including the vector fields P_μ are needed in order to calculate chiral corrections where B^* are included. From Eqs. (5), (7) and (54) the renormalization invariant bag parameter can be extracted. Anticipating the results of the two next sections, it can be written in the form:

$$\hat{B}_{B_q} = \frac{3}{4}\widetilde{b}\left[1+\frac{1}{N_c}\left(1-\delta_G^B(1+\frac{\tau_\chi^G}{32\pi^2 f^2})\right)+\frac{\tau_b}{m_b}+\left(1+\frac{1}{N_c}\right)\frac{\tau_\chi}{32\pi^2 f^2}\right] \;, \tag{58}$$

where

$$\widetilde{b} \;=\; b(m_b)\,\left[\frac{C_1-C_2}{(C_\gamma+C_v)^2}\right]_{\mu=\Lambda_\chi} \;. \tag{59}$$

We find from (54) the parameter due to genuine non-factorizable effects:

$$\delta_G^B \;=\; N_c\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{\beta_B}{\alpha_H^2}\;\left[\frac{2C_1}{C_1-C_2}\right]_{\mu=\Lambda_\chi} \;. \tag{60}$$

Note that this parameter is formally of order $(N_c)^0$ and is positive, which means that this non-factorizable contribution reduces the value of $\hat{B}$ according to (58). Thus we are qualitatively in agreement with [5], where a negative contribution to the bag factor from gluon condensate effects is found.

Using the relation between α_H and f_B in Eq. (48) and the expression value for G_H in Eq. (33), we may also write:

$$\delta_G^B \;=\; \frac{N_c\langle\frac{\alpha_s}{\pi}G^2\rangle}{32\pi^2 f^2 f_B^2}\,\frac{m}{M_B}\,\rho\left\{1+\frac{4\pi}{N_c}\left(\frac{f}{m}\right)^2+\frac{8\pi^2}{N_c^2}\left(\frac{f}{m}\right)^4\right\}\left[\frac{C_1}{C_1-C_2}\right]_{\mu=\Lambda_\chi} \;. \tag{61}$$

Numerically, f and f_B are of the same order of magnitude, and δ_G^B is therefore suppressed like m/M_B compared to the corresponding quantity

$$\delta_G^K \;=\; N_c\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{32\pi^2 f^4} \;, \tag{62}$$

for $K-\overline{K}$ mixing. However, one should note that f_B scales as $1/\sqrt{M_B}$ within HQEFT, and therefore δ_G^B is still formally of order $(m_b)^0$.

The formula (58) is a generalization of a similar formula found for $K-\overline{K}$ mixing [6]. The quantities τ_b and τ_χ^G will be calculated in the next sections, while τ_χ is known from previous work [29]. More specific, the quantity τ_b, to be calculated in the next section, has dimension $(mass)^1$ and depend on hadronic parameters calculated within the HLχQM. Similarly, the quantity τ_χ contains the chiral corrections to the bosonized versions of $Q_{1,2}$ to be presented in section VI. The quantity τ_χ^G contains the chiral corrections proportional to $\langle\frac{\alpha_s}{\pi}G^2\rangle$ and the counter-terms ω_β and ω_1.

5 $1/m_Q$ corrections

The $1/m_Q$ corrections have been defined in Eq. (14-17). In the HLχQM we only need to consider (14) and (17). This is due to the fact that when we are considering terms in the effective Lagrangian for $B-\overline{B}$ mixing the external particles carry no redundant momenta [7]. (In other words, the B-mesom momenta are $p_B = M_B v$). Hence the operators in (15) and (16) will give zero contribution.

The operator in Eq. (14) can be written on the form

$$X_1 = 2\sum_{j=1}^{3} \overline{q}\,\Gamma_j\, i\, D_\alpha\, Q_v^{(+)}\; \overline{q}\,\Theta_j\, Q_v^{(-)} \;+\; 2\sum_{j=4}^{6} \overline{q}\,\Gamma_j\, Q_v^{(+)}\; \overline{q}\,\Theta_j\; i\, D_\alpha\, Q_v^{(-)}\,, \qquad (63)$$

where Γ^α, Θ are defined:

$$\begin{array}{ll} \Gamma_1 = R\,\gamma^\alpha & \Theta_1 = R \\ \Gamma_2 = R\,g^{\mu\alpha} & \Theta_2 = R\,\gamma_\mu \\ \Gamma_3 = -i\,\varepsilon^{\lambda\alpha\nu\rho}\,v^\lambda\,R\,\gamma^\nu & \Theta_3 = R\,\gamma^\rho \\ \Gamma_4 = R & \Theta_4 = R\,\gamma^\alpha \\ \Gamma_5 = R & \Theta_5 = R\,g^{\mu\alpha} \\ \Gamma_6 = -i\,\varepsilon^{\lambda\alpha\nu\rho}\,v^\lambda\,R\,\gamma^\rho & \Theta_6 = R\,\gamma^\nu\,, \end{array} \qquad (64)$$

where D_α is the covariant derivative containing the gluon field. Note that the operator X_1 is Fierz symmetric [15]. We bosonize X_1 in the same way as $Q_{1,2}$.

Some two-quark operators appearing in (63) are already studied in [7] when calculating $1/m_b$ corrections to f_B. We use those results when bosonizing X_1, and the result can be written:

$$\begin{aligned} & X_1 \to X_1^{\rm bos} = \\ & \sum_{i=1}^{3}\Big\{2(1+\frac{1}{N_c})\frac{\alpha_H}{2}Tr\left[\xi^\dagger\Theta_i\,H_v^{(-)}\right]\;\frac{1}{2}Tr\left[\xi^\dagger\Gamma_i\,H_v^{(+)}\,(\alpha_3^\gamma\gamma^\alpha+\alpha_3^v v^\alpha)\right] \\ & +4\beta_1 Tr\left[\xi^\dagger\Theta_i H_v^{(-)}\,(-\beta_2\left\{\sigma^{\mu\nu},\gamma\cdot v\right\}+\beta_4\sigma^{\mu\nu})\Big\}\right. \\ & \left. Tr\left[\xi^\dagger\Gamma_i H_v^{(+)}\,(\beta_3 D_{\mu\nu\alpha}+2m\beta_2\sigma_{\mu\nu}v_\alpha)\right\}\right\} \\ & +\sum_{i=4}^{6}\Big\{2(1+\frac{1}{N_c})\frac{\alpha_H}{2}Tr\left[\xi^\dagger\Gamma_i\,H_v^{(-)}\,(\alpha_3^\gamma\gamma^\alpha-\alpha_3^v v^\alpha)\right]\;\frac{1}{2} \\ & Tr\left[\xi^\dagger\Theta_i\,H_v^{(+)}\right] \\ & +4\beta_1 Tr\left[\xi^\dagger\Gamma_i H_v^{(-)}\,(\beta_3 D_{\mu\nu\alpha}-2m\beta_2\sigma_{\mu\nu}v_\alpha)\right] \\ & Tr\left[\xi^\dagger\Theta_i H_v^{(+)}\,(\beta_2\left\{\sigma^{\mu\nu},\gamma\cdot v\right\}+\beta_4\sigma^{\mu\nu})\right]\rangle\Big\}\quad, \end{aligned} \qquad (65)$$

where $D_{\mu\nu\alpha} \equiv \{\sigma_{\mu\nu},\gamma_\beta\}\,(g_{\alpha\beta}-v_\alpha v_\beta)$. The second and fourth lines are genuinely non-factorizable. The α's and β's are hadronic parameters calculated within the HLχQM, and are given in Appendix B. Evaluating the sums and traces in Eq. (65) we arrive at:

$$\begin{aligned} X_1^{\rm bos} \quad = \quad & \left\{\alpha_H\alpha_3^\gamma(1+\frac{1}{N_c})+\langle\frac{\alpha_s}{\pi}G^2\rangle\beta_B^{(2)}\right\} \\ & \left(-P_i^{(-)\mu}\Sigma_{ii}^\dagger P_{i\mu}^{(+)}+3P_{5i}^{(-)}\Sigma_{ii}^\dagger P_{5i}^{(+)}\right)\;, \end{aligned} \qquad (66)$$

where $\beta_B^{(2)}$ is a combination of the β_i's and can be written

$$\beta_B^{(2)} \equiv \frac{\pi}{4N_c}(1-g_{\mathcal{A}})\left(1+\frac{4\pi}{3N_c}\left(\frac{f}{m}\right)^2\right) \ . \tag{67}$$

The bosonating of the nonlocal operators is rather straight forward in this model. The result for the factorizable part of the non local operators can be found in [7] in the calculation of f_B:

$$\begin{aligned}\sum_{i=1}^{4}\frac{A_i S_i^{\mathrm{Fact}}}{m_b} \rightarrow \qquad & -\left(1+\frac{1}{N_c}\right)\frac{\alpha_H}{m_b G_H}\left(\mu_\pi^2 - d_{\mathcal{M}}\frac{\mu_G^2}{3}\right) \\ & \times\left[C_1 P_i^{(-)\mu}\Sigma_{ii}^\dagger P_{i\mu}^{(+)} + (C_1 - C_2)P_{5i}^{(-)}\Sigma_{ii}^\dagger P_{5i}^{(+)}\right] \ . \end{aligned} \tag{68}$$

The result for the nonfactorizable part of the operators is:

$$\begin{aligned} & \sum_{i=1}^{4}\frac{A_i S_i^{\mathrm{Nfact}}}{m_b} \rightarrow \\ & \frac{1}{m_b}\langle\frac{\alpha_s}{\pi}G^2\rangle\beta_K\left((C_1-\frac{1}{3}C_2)P_i^{(-)\mu}\Sigma_{ii}^\dagger P_{i\mu}^{(+)} + C_1 P_{5i}^{(-)}\Sigma_{ii}^\dagger P_{5i}^{(+)}\right) \\ + \qquad & \frac{1}{m_b}\langle\frac{\alpha_s}{\pi}G^2\rangle C_M\left(C_1\beta_M^{(1)} + C_2\beta_M^{(2)}\right)\left[-P_i^{(-)\mu}\Sigma_{ii}^\dagger P_{i\mu}^{(+)} + 3P_{5i}^{(-)}\Sigma_{ii}^\dagger P_{5i}^{(+)}\right] \ , \end{aligned} \tag{69}$$

where the quantities β_K and $\beta_M^{(1,2)}$'s are given in Appendix B.

We need f_B which has been calculated in [7] to $1/m_b$:

$$\begin{aligned} & f_H\sqrt{M_H} = \alpha_H(C_\gamma + C_v)\left(1+\frac{\kappa_b}{m_b}+\frac{\kappa_\chi}{32\pi^2 f^2}\right) \ , \ \text{where:} \\ & \kappa_b = -\frac{(\varepsilon_1 - 6\varepsilon_2)}{2} + \frac{(B_\gamma\alpha_3^\gamma + B_v\alpha_3^v)}{2\alpha_H(C_\gamma - C_v)} - \frac{(\mu_\pi^2-\mu_G^2)}{G_H\alpha_H} \\ & \kappa_{\chi_d} = -\frac{11}{18}\left\{-m_K^2(1+g_{\mathcal{A}}{}^2) + m_K^2(\ln\frac{m_K^2}{\mu^2}+\frac{2}{11}\ln\frac{4}{3})(1+3g_{\mathcal{A}}{}^2)\right\} \ , \\ & \kappa_{\chi_s} = -\frac{13}{9}\left\{-m_K^2(1+g_{\mathcal{A}}{}^2) + m_K^2(\ln\frac{m_K^2}{\mu^2}+\frac{4}{13}\ln\frac{4}{3})(1+3g_{\mathcal{A}}{}^2)\right\} \\ & +\frac{\omega_1 32\pi^2 f^2}{\alpha_H}m_s \ , \end{aligned} \tag{70}$$

where B_γ and B_v are sums of Wilson coefficients. The contribution to the bag parameter from $1/m_b$ corrections can now be extracted (see Eq. (58)):

$$\begin{aligned}\tau_b = \qquad & \left(1+\frac{1}{N_c}\right)\left\{\frac{\alpha_3^\gamma}{\alpha_H}\left(\frac{6B_1}{C_1-C_2}-\frac{B_\gamma}{C_\gamma+C_v}\right)-\frac{\alpha_3^v}{\alpha_H}\frac{B_v}{(C_\gamma+C_v)}\right\} \\ & +\frac{6C_1}{(C_1-C_2)\alpha_H^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\left\{\frac{B_1}{C_1}\beta_B^{(2)}+\frac{\beta_K}{3}+C_M\beta_M^{(1)}+\frac{C_2C_M}{C_1}\beta_M^{(2)}\right\} \ . \end{aligned} \tag{71}$$

It should be noted that $1/m_b$ corrections increases $\hat{B}$, in agreement with [15].

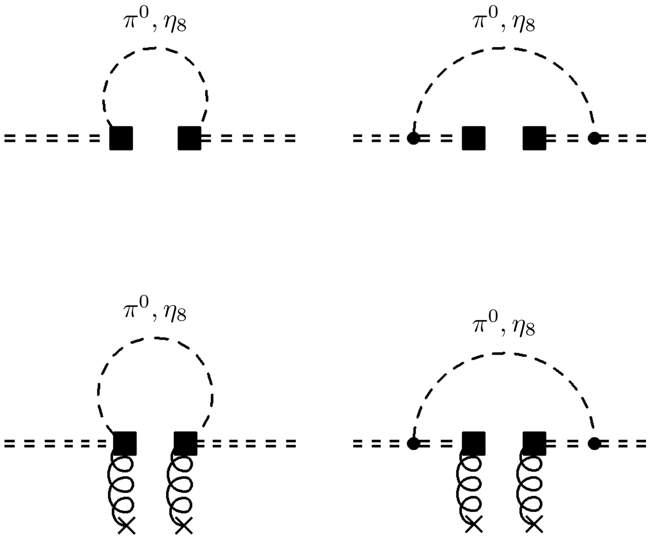


Fig. 4. Diagrams contributing to the bag parameter

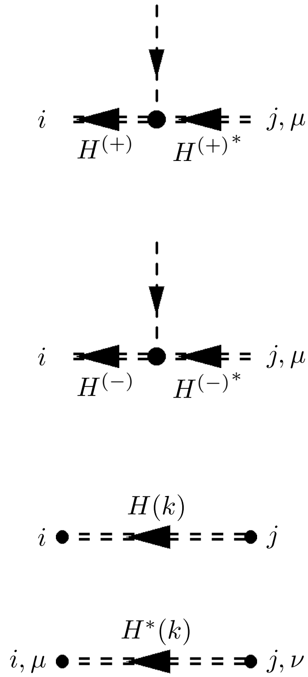
6 Chiral corrections

We will only consider chiral corrections to $Q_{1,2}$ in Eqs. (12) and (13). Adding chiral corrections to operators proportional to $1/m_Q$ will be considered as higher order. The chiral corrections to the bag parameter have been considered in [29]. Some of the corrections are simply corrections to f_{B_q} [30, 31, 32]. The diagrams shown in Fig. 4 are those which are genuinely non-factorizable, i.e. they are not included in chiral corrections to f_{B_q}.

The chiral corrections (τ_χ) to the bag parameter can then be written:

$$\begin{aligned}
\tau_\chi = \quad & d_\chi \left\{ -\frac{2}{9} m_K^2 \ln\left(\frac{4m_K^2}{3\mu^2}\right) - \frac{2}{9} m_K^2 \right. \\
& + \frac{C_1}{C_1 - C_2} {g_{\mathcal{A}}}^2 \left((\frac{2}{3} m_K^2 - \Delta^2) \ln\left(\frac{4m_K^2}{3\mu^2}\right) \right. \\
& \left.\left. - \frac{8}{9} m_K^2 + \frac{8}{3}\Delta^2 (\, 2 - 3F(\Delta/m_\eta)\,) \right) \right\} \;, \qquad (72)
\end{aligned}$$

$$\begin{aligned}
\tau_\chi^G = \quad & d_\chi \left\{ -\frac{2}{9} m_K^2 \ln\left(\frac{4m_K^2}{3\mu^2}\right) - \frac{2}{9} m_K^2 \right. \\
& + \frac{C_1 - C_2/3}{C_1} {g_{\mathcal{A}}}^2 \left((\frac{2}{3} m_K^2 - \Delta^2) \ln\left(\frac{4m_K^2}{3\mu^2}\right) \right. \\
& \left.\left. - \frac{8}{9} m_K^2 + \frac{8}{3}\Delta^2 (\, 2 - 3F(\Delta/m_\eta)\,) \right) \right\} \\
& - d_s \left(\frac{\omega_\beta}{\beta_B} + 2\frac{\omega_1}{\alpha_B} \right) 32\pi^2 f^2 \, m_s \;, \qquad (73)
\end{aligned}$$



$$-2\,i\,g_{\mathcal{A}}\,k_\mu\,\Pi_{ij}$$

$$2\,i\,g_{\mathcal{A}}\,k_\mu\,\Pi_{ij}$$

$$\frac{i}{2(v\cdot k-\Delta)}\delta_{ij}$$

$$-\frac{i(g^{\mu\nu}-v^\mu v^\nu)}{2(v\cdot k-\Delta)}\delta_{ij}$$

Fig. 5. Feynman rules for the strong sector, Π is given in Eq. (22)

where we have ignored the pion mass and used the mass relations $m_{\eta_8}^2 = 4m_K^2/3$. The function $F(x)$ is defined in Eq. (83) and:

$$d_\chi = \begin{cases} 1 & \text{for} \quad B_d \\ 4 & \text{for} \quad B_s \end{cases} \qquad \text{and} \quad d_s = \begin{cases} 0 & \text{for} \quad B_d \\ 1 & \text{for} \quad B_s \end{cases} \tag{74}$$

If one ignores the counter-term given by ω_β, and take the limit $\Delta \equiv M_H^* - M_H \to 0$, we obtain the same result as in [29]. For the bare coupling constant f we will use the value f=86 MeV [32]. The Feynman rules for chiral loops are given in Fig. 5.

7 Numerical results

The model dependent parameters of the HLχQM was fixed in [7] by using various constraints. For instant, the constituent light quark mass was determined to be $m = 220 \pm 30$ MeV. Using the parameters from [7], we obtain (using $\Delta =$

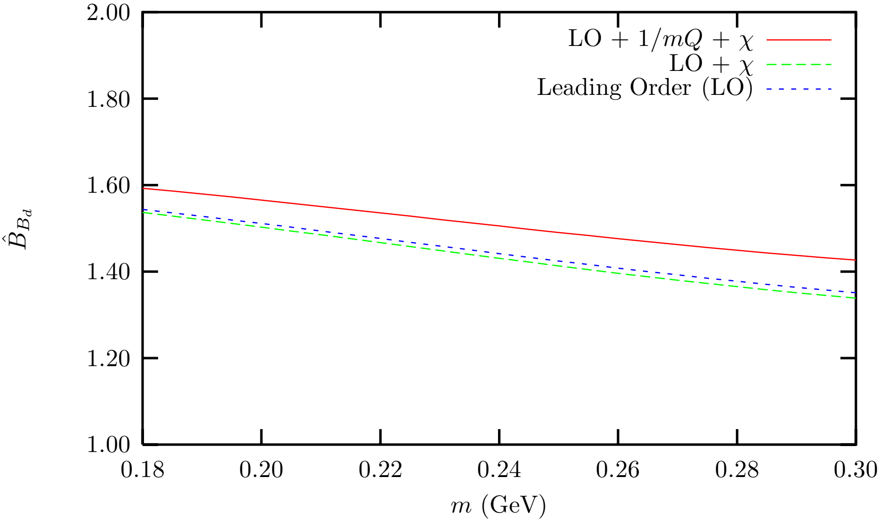


Fig. 6. The bag parameter $\hat{B}$ for B_d

$M_H^* - M_H = 0.025$ GeV):

$$\begin{aligned}
\tau_b &= (0.26 \pm 0.04)\,\mathrm{GeV} & \delta_G^B &= (0.5 \pm 0.1) \\
\tau_{\chi_d} &= -(0.02 \pm 0.01)\,\mathrm{GeV}^2 & \tau_{\chi_s} &= -(0.10 \pm 0.04)\,\mathrm{GeV}^2 \\
\tau_{\chi_d}^G &= -(0.03 \pm 0.01)\,\mathrm{GeV}^2 & \tau_{\chi_s}^G &= (0.12 \pm 0.06)\,\mathrm{GeV}^2 \\
\hat{B}_{B_d} &= 1.53 \pm 0.05 & \hat{B}_{B_s} &= 1.48 \pm 0.08 \\
f_{B_d} &= (170 \pm 25)\,\mathrm{MeV} & f_{B_s} &= (180 \pm 25)\,\mathrm{MeV} \\
f_{B_d}\sqrt{\hat{B}_{B_d}} &= (215 \pm 30)\,\mathrm{MeV} & f_{B_s}\sqrt{\hat{B}_{B_s}} &= (225 \pm 30)\,\mathrm{MeV} \\
\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} &= 1.05 \pm 0.01 & \frac{f_{B_s}}{f_{B_d}} &= 1.08 \pm 0.02
\end{aligned} \tag{75}$$

The decay constants f_{B_d} and f_{B_s} were also given in [7], but are listed also here for completeness. (Note, however, that the values are slightly different, because in [7] we did not distinguish f_π from the bare coupling f.) The values for the bag parameter $\hat{B}$ are in agreement with lattice calculations [3, 4]. A plot of $\hat{B}$ as a function of the constituent quark mass m is shown in Fig. 6 and 7. We observe that the values of $\hat{B}$ are fairly stable over a large variation of light quark constituent mass m. Especially this is the case for B_d. From $m = 180$ MeV and $m = 300$ MeV the bag factors only changes with 10%. We note that $1/m_b$ corrections are small.

The values for the f_B's and especially for the ratio f_{B_s}/f_{B_d} (and ξ) in (75) are a bit low [3, 33]. There might be at least three reasons for this. First, concerning the absolute value for f_B's, they dependent significantly on the value of the quark condensate, as seen from Eqs. (45) and (48). In [7] we used the "standard" value $\langle \overline{q}q \rangle = (-240\,\mathrm{MeV})^3$, without any uncertainty. It could be argued that we should

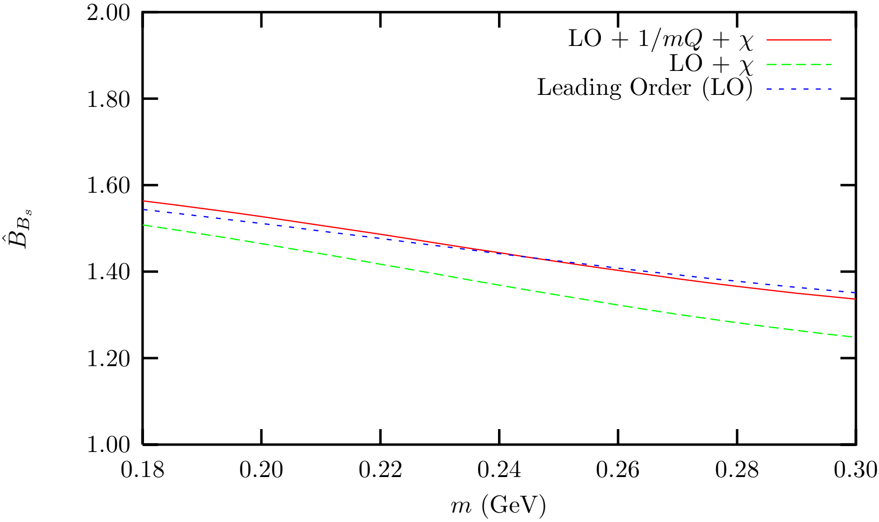


Fig. 7. The bag parameter $\hat{B}$ for B_s

have used an uncertainty of 10 MeV, say, for $\langle \bar{q}q \rangle^{1/3}$, although the wide range 190 to 250 MeV used for m will to some extent compensate for this. Second, it might be that our expansion within the HLχQM overestimates the counter-term ω_1 which reduces f_{B_s}. However, neglecting this counter-term would give the high value $f_{B_s}/f_{B_d} \simeq 1.3$. Third, our value for the axial pion coupling $g_{\mathcal{A}}$ in (28) might be too low. In [7] we used input from QCD sum rules [34] both in the B- and D-sectors. Alternatively, we may use the experimental value for the effective coupling ${g_{\mathcal{A}}}^{H^*H\pi} = 0.59 \pm 0.09$ in the D-sector [35], giving almost the same bare coupling $g_{\mathcal{A}} = 0.59 \pm 0.04$. Using this bare coupling also in the B-sector (instead of $g_{\mathcal{A}} = 0.42 \pm 0.06$ in [7]), and in addition $\langle \bar{q}q \rangle^{1/3} = (-240 \pm 10)\,\mathrm{MeV}$, we obtain an alternative set of values:

$$
\begin{aligned}
\tau_b &= (0.25 \pm 0.04)\,\mathrm{GeV} & \delta_G &= (0.5 \pm 0.2) \\
\tau_{\chi d} &= -(0.06 \pm 0.01)\,\mathrm{GeV}^2 & \tau_{\chi_s} &= -(0.25 \pm 0.04)\,\mathrm{GeV}^2 \\
\tau_{\chi d}^G &= -(0.07 \pm 0.01)\,\mathrm{GeV}^2 & \tau_{\chi_s}^G &= (0.2 \pm 0.2)\,\mathrm{GeV}^2 \\
\hat{B}_{B_d} &= 1.51 \pm 0.09 & \hat{B}_{B_s} &= 1.37 \pm 0.14 \\
f_{B_d} &= (190 \pm 50)\,\mathrm{MeV} & f_{B_s} &= (210 \pm 70)\,\mathrm{MeV} \\
f_B\sqrt{\hat{B}_{B_d}} &= (240 \pm 70)\,\mathrm{MeV} & f_{B_s}\sqrt{\hat{B}_{B_s}} &= (260 \pm 90)\,\mathrm{MeV} \\
\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} &= 1.08 \pm 0.07 & \frac{f_{B_s}}{f_{B_d}} &= 1.14 \pm 0.07
\end{aligned}
\tag{76}
$$

We observe that the value for f_{B_s}/f_{B_d} in (76) is close to the standard one.

To conclude, we have calculated the bag parameter $\hat{B}$ for the B_d and B_s mesons. Combining our two alternative sets of values (and consider the range of values) we find $\hat{B}_{B_d} = 1.51 \pm 0.09$ and $\hat{B}_{B_s} = 1.40 \pm 0.16$. The value for $\hat{B}_{B_s}$ is more sensitive to chiral loops and counter-terms, and therefore the uncertainty is bigger.

In principle, $\hat{B}$ is renormalization invariant (μ independent). This cannot be shown within our approach. By construction, perturbative QCD within HQEFT, the HLχQM and chiral perturbation theory are matched at the scale Λ_χ. However, we have a reasonable good matching numerically as in [23]. Varying the renormalization scale $\mu = \Lambda_\chi$ in the range 0.8 GeV to 1 GeV, the bag parameters only change with 6%. Moreover, like in [6], the formula (58) nicely shows the various parts building up the total result for $\hat{B}$.

A Loop integrals

The divergent integrals entering in the bosonization of the HLχQM are defined:

$$I_1 \equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2 - m^2} \tag{77}$$

$$I_{3/2} \equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{(v\cdot k)(k^2 - m^2)} \tag{78}$$

$$I_2 \equiv \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m^2)^2} \tag{79}$$

The integrals needed in the calculation of chiral corrections to the bag parameter are:

$$\begin{aligned} L_{1,1}^{m,\Delta} &= \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m^2)(v\cdot k - \Delta)} \\ &= \frac{-i\Delta}{8\pi^2} \left(\frac{1}{\bar{\varepsilon}} - \ln(m^2) + 2 - 2F(m/\Delta) \right) \end{aligned} \tag{80}$$

$$\begin{aligned} &\int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - m^2)(v\cdot k - \Delta)} = A\, g^{\mu\nu} + B\, v^\mu v^\nu \\ &A = \frac{1}{d-1} \int \frac{d^dk}{(2\pi)^d} \frac{k^2 - (v\cdot k)^2}{(k^2 - m^2)(v\cdot k - \Delta)} \\ &= \frac{i\Delta}{16\pi^2} \left\{ (-\frac{1}{\bar{\varepsilon}} + \ln(m^2) - 1)(m^2 - \frac{2}{3}\Delta) - \frac{4}{3}F(m/\Delta)(\Delta^2 - m^2) \right. \\ &\left. -\frac{4}{3}(m^2 - \frac{5}{6}\Delta^2) \right\} \end{aligned} \tag{81}$$

$$\begin{aligned} &B = -A + \int \frac{d^dk}{(2\pi)^d} \frac{(v\cdot k)^2}{(k^2 - m^2)(v\cdot k - \Delta)} \\ &= \frac{-i\Delta}{16\pi^2} \left\{ (-\frac{1}{\bar{\varepsilon}} + \ln(m^2) - 1)(2m^2 - \frac{8}{3}\Delta) - \frac{4}{3}F(m/\Delta)(4\Delta^2 - m^2) \right. \\ &\qquad \left. -\frac{4}{3}(m^2 - \frac{7}{3}\Delta^2) \right\} \end{aligned} \tag{82}$$

where:

$$F(x) = \begin{cases} -\sqrt{x^2-1}\tan^{-1}(\sqrt{x^2-1}) & x > 1 \\ \sqrt{1-x^2}\tanh^{-1}(\sqrt{1-x^2}) & x < 1 \end{cases} \tag{83}$$

In the case of $\Delta > m$ we have ignored an analytic real part in (80). Equation (80) coincides with the one obtained in [30] however Eq. (82) differs by a factor $-2/3(m^2 - 2/3\Delta^2)$ inside the parenthesis of the expressions for A and B. This is presumably due to the factor $1/(d-1) = (1-2/3\varepsilon)/3$ in A.

B Some detailed expressions for hadronic parameters

The parameters of Eq. (65) are:

$$\begin{aligned}
\alpha_3^\gamma &\equiv \frac{m}{3}\alpha_H + \frac{G_H}{6}\langle\,\overline{q}q\,\rangle \\
\alpha_3^v &\equiv \frac{m}{3}\alpha_H + \frac{2}{3}G_H\langle\,\overline{q}q\,\rangle \\
\beta_1 &\equiv \frac{G_B^2\pi^2}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle \\
\beta_2 &\equiv -\frac{f^2}{4m^2N_c} \\
\beta_3 &\equiv -\frac{\delta g_{\mathcal{A}}}{4G_B^2N_c} \\
\beta_4 &\equiv \frac{1}{8\pi}
\end{aligned} \tag{84}$$

The $\beta_{K,M}^{(1,2)}$'s in (69) are given by:

$$\begin{aligned}
\beta_K^{(1)} &= \frac{m}{256\pi^2}G_B^2\left\{1+\frac{4}{\pi}-\frac{8\pi}{N_c}\left(\frac{f}{m}\right)^2(1+\frac{1}{\rho}-\pi)-\frac{32\pi^2}{N_c^2}\left(\frac{f}{m}\right)^4\right. \\
&\qquad\left. -C_K\left[\frac{8\pi}{N_c}\left(\frac{f}{m}\right)^2+\frac{16\pi^2}{N_c^2}\left(\frac{f}{m}\right)^4\right]\right\}
\end{aligned} \tag{85}$$

$$\beta_M^{(1)} = -\frac{\pi^2}{12N_c^2}\left(\frac{f}{m}\right)^2 \tag{86}$$

$$\beta_M^{(2)} = \frac{\pi}{24N_c}\left\{1+\frac{2\pi}{N_c}\left(\frac{f}{m}\right)^2\right\} \tag{87}$$

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